The Lifetime of Carbon-11

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We measured the lifetime of Carbon-11 to be $\tau = 1761.3 \pm 1.4 \text{s}$ corresponding to a half life of $20.347(16) \text{ min}$. Our measured value is in agreement with the accepted half-life of $20.334(24) \text{ min}$\textsuperscript{2}. The Carbon-11 was produced via the reaction $^{11}\text{B} + p \rightarrow n + ^{11}\text{C}$ using 6 MeV protons generated by a tandem Van De Graaff accelerator.

I. OVERVIEW

Carbon-11 can be produced by bombarding Boron with high energy protons through the following reaction: $^{11}\text{B} + p \rightarrow n + ^{11}\text{C}$. The reaction’s cross section has a maximum when the energy of the protons is near 6 MeV, so we used that energy in our experiment.\textsuperscript{3} (By comparison, the Coulomb barrier for $^{11}\text{B}$ is 1.62 MeV.\textsuperscript{5})

Carbon-11 decays 99.79(4)% of the time via positron emission ($^{11}\text{C} \rightarrow \beta^+ + e^- + ^{11}\text{B}$) and .21(4)% of the time via K-capture ($^{11}\text{C} \rightarrow ^{11}\text{B} + \gamma + e^{-\text{auger}}$).\textsuperscript{61}

To measure the lifetime, we used a coincidence circuit to detect gamma rays with energies of 511KeV. These gamma rays are produced by the anihilation of positrons emitted by the $^{11}\text{C}$ source. The two gamma rays are emitted in exactly opposite directions to conserve momentum. Two NaI detectors and photomultiplier tubes were wired to discriminators and into coincidence circuit.

II. PRELIMINARY CALIBRATION

The “resolving time” of the coincidence circuit, denoted by $\Delta t$, is the time maximum separation between the leading edge of the first signal and the trailing edge of the second signal whereby the two signals are still considered coincident. For our logic pulses of 50ns, the resolving time was set to 100ns.

To test and calibrate our circuit we used a 1 $\mu\text{Ci}$ Na-22 source which also decays by positron emission 90% of the time to $^{22}\text{Ne}$.\textsuperscript{4} We were able to clearly observe the 511keV signal on the oscilloscope, and set our discriminator threshold accordingly. We also observed lower energy signals from gamma rays which had Compton scattered and higher energy 1.277 MeV gamma rays from the de-excitation of $^{22}\text{Ne}$. We next determined that 50ns was the optimal pulse width by varying the pulse width and looking at how the number of coincidences changed (see figure II).

Some of the counts by any coincidence circuit will be “accidentals”, due to the possibility of the random overlap of two uncorrelated pulses. The rate of accidentals can be easily derived by considering that during the course of a measurement, the coincidence circuit will be open a fraction of the total time $f = R_1 \Delta t$, where $R_1$ is the rate in channel 1. The rate of accidentals will be:

$$R_{acc} = R_2 f = R_1 R_2 \Delta t$$  \hspace{1cm} (1)

We measured our accidental rate experimentally by adding a $> 100$ns delay to one of the lines. We found an accidental rate of 2.8 per minute, corresponding to a resolving time of 200ns $\pm$ 48ns. This is twice larger than our predicted value of 100ns, indicating a larger accidental rate than we would have expected from the equation. (this should have been investigated / explained).

In addition to the accidental rate, there is also a background rate, which is due to things like a cosmic ray passing through both detectors. We measured the background rate to be at 0.783 $\pm$ 0.386 counts per minute, by removing the source for 1.7 hours. The background rate is negligibly small compared to our coincidence count rate for Na-22.

III. MEASUREMENT OF $^{11}\text{C}$’S LIFETIME

The rate of radioactive decay of $^{11}\text{C}$ is described by the equation

$$R(t) = R_0 e^{-t/\tau}$$  \hspace{1cm} (2)

where $\tau$ is the lifetime, which is closely related to the half-life: $t_{1/2} = \ln(2) \tau$
The function we used to describe the rate of decay is:

\[-R(t) = \frac{A}{\tau_1} e^{-t/\tau_1} + \frac{B}{\tau_2} e^{-t/\tau_2} + C\]  

(3)

We measured the number of coincidences in time periods of 200s to get our rate data. To perform our fit we integrated equation 3 over each of those time periods and compared to what was measured (see figure 3).

We found the Carbon-11 lifetime to be \(\tau_1 = 1761.3 \pm 1.4\)s corresponding to a half life of \(1220.8(97)\)s = 20.347(16) min. Our fit had a normalized \(\chi^2 = 1.32\), corresponding to a p-value of .0984. Our measured value is in agreement with the accepted value of 20.334(24) min.\(^2\)

We also found that \(\frac{B}{\tau_2} e^{-t/\tau_2}\) term had a much smaller amplitude (about 2% of the first term) and that \(\tau_2 = 938.65 \pm 55\)s. We believe the second exponential decay is due to the accidental rate, which scales as the square of the total rate. Therefore, the lifetime of the accidental rate should be half the lifetime of Carbon-11, as we observed. The relative amplitude of the accidental rate also compares nicely with a measurement of the accidental rate for \(^{11}\)C we took, where we found \(R_{\text{acc}}/R_{\text{total}} \approx .01\pm .005\). We also found \(C = .17(04)\)min\(^{-1}\) which is slightly smaller than the background rate we measured previously. The difference in the background term could be due to a small systematic error in the experiment. Adding a third exponential to the fit equation did not improve the fit, indicating that no other radioactive isotopes with similiar half lifes were interfering with our measurements.

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dertron travels at 12 m/s. Charge is held on metal bars, which are separated by plastic resistors. The laddertron picks up positive charge on the low voltage side and transports it to the center, where it is deposited on a smooth metal shell. The laddertron also picks up negative charge from the center and transports it back. Charge is transferred onto the belt by a device called an “inductor”, which requires voltages around 50kV. The built up charge travels to ground through three ‘columns’ which are broken into 200 segments. Adjacent segments are
The entire system, including the ion source, Van De Graaf, and bending and focusing magnets are controlled by LabView software, with the exception of the Van de Graaf 'inductors' and voltage readout.

A schematic diagram of the beam path is shown in figure 5. After emerging from the ionizer, the beam first passes through an Einzel lens, which uses electrostatic fields to focus the beam. Then the beam passes through a steerer and then the inflection magnet which selects out the proper mass with a precision of $\Delta m/m \approx 1/130$. Additionally, the vertical magnetic field of the magnet works to help focus the beam.

Next the beam passes through another steerer and electrostatic triplet lens. A triplet configuration ensures the system is anastigmatic. The beam then passes through the Van de Graaf and the “analyzing magnet” directs the beam towards the target chamber. The beam then passes through a steerer and another focusing triplet. The beam then passes through a large layer of concrete (to provide radiation protection) into the target room. The correct beamline is selected by the “switching magnet”. Our target was a small chunk of boron suspended in an aluminum frame. Before the target there is a magnetic quadrupole focuser and another X-Y steerer. We can tell that the target is being hit because then the current measured on the “frame” around the target is maximized and the current measured after the target is near zero.

There are also four Faraday cups (not shown in figure 5) which can be inserted pneumatically to stop the beam at four strategic locations. Each Faraday cup consists of a metal cup attached to an electrometer which can read the current of the beam hitting the cup. Also not shown is the beam profile monitor, which is capable of displaying the beam cross section while interfering minimally with the beam. The beam profile monitor consists of a thin grounded helical wire connected to a drive motor. The rotating wire passes through the beam horizontally and vertical during each rotation. A cylindrical collector around the wire/beam picks up electrons which are knocked off the wire by the beam. The resulting current signal can be translated into a 2D cross section of the beam.

3. Beam tuning & control

What we neglected to mention in the previous section is that the entire ion source is on a table which is held at a potential of -4MV. This high voltage is generated a 170 kV high voltage power supply connected to two Cockcroft-Walton like generators.

We used the Van de Graaff as a tandem accelerator, meaning that the high voltage potential is utilized twice. The $\text{H}^-$ ions hit a thin carbon foil in the center of the Van de Graaff and are converted into $\text{H}^+$ ions, which are then accelerated again.

REFERENCES

5.(Nuclear Reaction Video project. Q-value calculator, March 2012.